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# Technical Notes

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## Similarities Between Variable Stiffness Springs and Piezoceramic Switching Shunts

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### I. Introduction

IN 1984, Chen [1] noted that space structures were becoming very large in size and were being constructed out of lightweight material making the large space structures (LSS) quite flexible. Because most of these structures must either precisely point in a specific direction or maintain a specific shape, the structure's vibrations caused by external disturbances needed to be controlled. Most of the research at that time in controlling LSS vibrations concentrated on active damping, in which arrays of distributed sensors and actuators were attached to the structure. The actuators would apply controlling forces that were proportional to the velocity of the structure. However, Chen noted that for the LSS currently under development, the structures could not support distributed actuators and sensors. For example, one LSS under development was a 12-bladed, photon-propelled spacecraft. Each of the blades was 8 m wide by 4000 m long and 0.254e-2 mm thick and constructed out of an extra-lightweight material. This structure would be stiff to in-plane bending and torsional vibrations, but would be very susceptible to out-of-plane vibrations. Therefore, Chen proposed controlling the out-of-plane vibrations by controlling the in-plane tension of the structure. In other words, control the structure by a variable stiffness control method. He demonstrated the feasibility this type of control using a simple string numerical example.

Following Chen's lead, Onoda et al. [2] devised two types of variable stiffness structural members for LSS (see Fig. 1). The Type I structural member consists of two springs in parallel: a primary spring ($k$) and a secondary spring that can continuously vary its stiffness from zero to a maximum stiffness ($\Delta k_{\max}$). Therefore, the stiffness of a Type I member ($k^*$) can have a structural stiffness of

$$k \leq k^* \leq k + \Delta k_{\max} \tag{1}$$

The Type II structural member is also constructed of two springs in parallel except the variable spring is replaced with a spring that is either clamped or free. Therefore, the Type II member can vary its stiffness to a high stiffness state or to a low stiffness state by either clamping or releasing $\Delta k$ (see Fig. 1).

Note that in the limiting case, the Type I mechanical spring can act as a Type II mechanical spring by rapidly varying its secondary spring stiffness from either zero to maximum stiffness or maximum to zero stiffness. However, the mechanisms that change the stiffness of the Type I and II variable stiffness members are inherently different from each other.

Onoda et al. [2] devised single mode vibration control laws for both the Type I and II structural members. Using simple single degree of freedom systems (single modal models), they showed that to reduce the dynamic response of a system with a Type I stiffness member, it is best to switch $k^*$ from $k + \Delta k_{\max}$ to $k$ when $y\dot{y}$ is negative and then switch $k^*$ from $k$ to $k + \Delta k_{\max}$ when $y\dot{y}$ is positive. They also noted that switching the Type I member at incorrect times in the vibration cycle of the system could add energy into the system and cause system instabilities. To reduce the dynamic response of a system with a Type II stiffness member, they showed that it is best to briefly switch from a high stiffness state to a low stiffness state and then back to the high stiffness state when the system is at maximum/minimum modal displacement ($\dot{y} = 0$). Switching at incorrect times in the vibration cycle for a system with Type II member will not, however, add energy to the system and will not cause instabilities. For more information on Onoda et al.'s mechanical spring control laws or energy dissipation mechanisms see [2].

The preceding single mode vibration control laws for the Type I and II mechanical springs are quite similar to two of the piezoceramic real-time shunt switching vibration control techniques: state switching [3,4] and synchronized switching [5,6]. Therefore, it is prudent to ask how the type of shunt circuit coupled with a piezoceramic actuator determines the overall characteristics of that actuator. In this paper, piezoceramic actuators coupled with switching shunts are investigated to determine the similarities between mechanical and piezoceramic variable stiffness members. Also, a third type of variable stiffness structural member (Type III mechanical spring) is introduced.

### II. Mechanical Springs

#### A. Type I Variable Stiffness Member

Typically, the dynamics of a flexible structure can be described by the familiar $n$-dimensional set of second order linear differential equations. Therefore the dynamics of a structure with a Type I variable stiffness member can be described as

$$[M]\ddot{x} + [K^*]x = [F]f \tag{2}$$

where $[M]$ is the mass matrix, $[F]$ is the input influence matrix, $x$ is an $n$-vector of generalized displacements, $f$ is an $n$-vector of generalized forces, and $[K^*]$ is the stiffness matrix that varies from

$$[K] \quad \text{to} \quad [K + \Delta K_{\max}] \tag{3}$$

where $[K]$ is the nominal low stiffness state matrix and $[K + \Delta K_{\max}]$ is the maximum high stiffness state matrix. This system can also be described in modal coordinates as:

$$[m]\ddot{y} + [k + \Delta k_{\max}]y = [\phi]^T[\Delta K][\phi]y + [\phi]^T[F]f \tag{4}$$

where $[m]$ and $[k + \Delta k_{\max}]$ are the diagonal modal mass and modal

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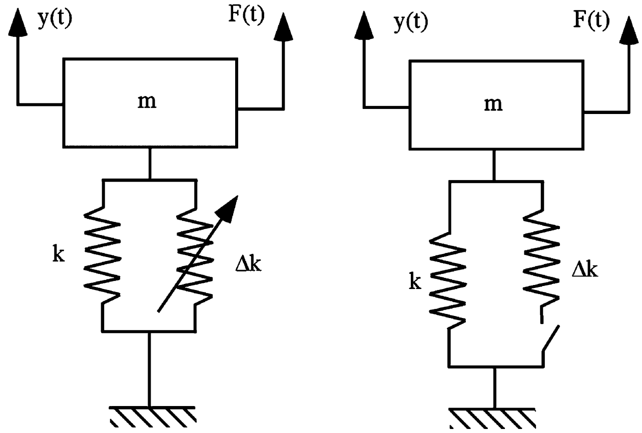


**Fig. 1 Example of Type I and II modal systems.**

high stiffness state matrices, respectively, $y$ is an $n$-vector of modal coordinates, $[\phi]$ is the high stiffness state modal matrix, and $[\Delta K]$ is the difference stiffness matrix (i.e., $[K + \Delta K_{\max}] - [K^*]$).

From Onoda et al. [2], for effective single mode vibration control, the Type I structural member should either be in a high stiffness state $[K + \Delta K_{\max}]$ or a low stiffness state $[K]$. Therefore, the system using this control law can be described in modal coordinates as

$$[m]\ddot{y} + [k + \Delta k_{\max}]y = (1 - s)[\phi]^T[\Delta K_{\max}][\phi]y + [\phi]^T[F]f \tag{5}$$

where $[\Delta K_{\max}]$ is the maximum difference stiffness matrix (i.e., $[K + \Delta K_{\max}] - [K]$), and $s$ changes between 0 and 1 depending on the control state:

$$s = 0 \rightarrow \text{Low Stiffness State}$$

$$s = 1 \rightarrow \text{High Stiffness State}$$

### B. Type II Variable Stiffness Member

A flexible structure with a Type II variable stiffness member can be described as

$$[M]\ddot{x} + [K]x + [\Delta K](x - x_o) = [F]f \tag{6}$$

when the system is in a high stiffness state (clamped spring), and

$$[M]\ddot{x} + [K]x = [F]f \tag{7}$$

when the system is in a low stiffness state (freed spring). $[M]$, $[K]$, $[F]$, $x$, and $f$ are the same as defined for the Type I structural member, $[\Delta K]$ is the difference stiffness matrix (i.e., $[K + \Delta K] - [K]$), and $x_o$ denotes the generalized displacements of the system at the time the system switches to the high stiffness state. Combining Eqs. (6) and (7) and converting into modal coordinates, the system can be described as

$$\begin{aligned}&[m]\ddot{y} + [k + \Delta k]y\\ &\quad = s[\phi]^T[\Delta K][\phi]y_o + (1 - s)[\phi]^T[\Delta K][\phi]y + [\phi]^T[F]f\end{aligned} \tag{8}$$

where $[m]$, $[F]$, $[\phi]$, $y$, $f$, and $s$ are the same as defined for the Type I structural member, $[k + \Delta k]$ is the diagonal modal high stiffness state matrix, and $y_o$ is an $n$-vector of modal coordinates at the time the system switches to a high stiffness state.

## III. Piezoceramic Springs

Piezoceramic actuators with resonant shunt circuits have been used over two decades to add damping to vibrating structures. The resonant shunt typically consists of a resistor and inductor in either parallel or series and is tuned to add damping to one structural mode. Recently, researchers have been using shunt circuits that can be switched to and from the piezoceramic actuators in real time (see Fig. 2). Three of these piezoceramic shunts will be used in this paper to investigate the affects they have on the overall characteristics of piezoceramic actuators: a capacitance ladder shunt (used in a tunable piezoceramic vibration absorber [7]), a switching ground shunt (used with the state switching technique), and a switching resistor/inductor shunt (used with the synchronized switching technique).

### A. Type I Piezoceramic Spring: Capacitance Ladder Shunt

It can be shown that a flexible structure that is coupled with a piezoceramic actuator can be described as [8]

$$[M]\ddot{x} + [K + K^{\mathrm{sc}}]x = \Theta\frac{Q^{\mathrm{app}}}{C_p^S} - \frac{1}{C_p^S}\Theta\Theta^T x + [F]f \tag{9}$$

where $[M]$, $[F]$, $x$, and $f$ are the same as defined for a Type I structure, $[K + K^{\mathrm{sc}}]$ is the low stiffness state matrix, which is a combination of the system's structural stiffness and the piezoceramic actuator's short circuit (low) stiffness, $\Theta$ is the electromechanical coupling vector, $C_p^S$ is the total capacitance of the piezoceramic actuator, and $Q^{\mathrm{app}}$ is the charge applied to the piezoceramic actuator. This can be written in modal coordinates as

$$[m]\ddot{y} + [k + k^{\mathrm{oc}}]y = [\phi]^T\Theta\frac{Q^{\mathrm{app}}}{C_p^S} + [\phi]^T[F]f \tag{10}$$

where

$$\begin{aligned}&[\Delta K] = \frac{1}{C_p^S}\Theta\Theta^T\\ &[(k + k^{\mathrm{sc}}) + \Delta k] = [\phi]^T\{[K + K^{\mathrm{sc}}] + [\Delta K]\}[\phi] = [k + k^{\mathrm{oc}}]\end{aligned} \tag{11}$$

and $[k + k^{\mathrm{oc}}]$ is the diagonal modal high stiffness state (open circuit) matrix. It is easy to show that the applied charge to the piezoceramic actuator ($Q^{\mathrm{app}}$) with a capacitance ladder is

$$Q^{\mathrm{app}} = \frac{C^{\mathrm{ladder}}}{C^{\mathrm{ladder}} + C_p^S}\Theta^T[\phi]y \tag{12}$$

where $C^{\mathrm{ladder}}$ is the total capacitance of the shunt. Substituting Eq. (12) into Eq. (10) yields

$$[m]\ddot{y} + [k + k^{\mathrm{oc}}]y = \frac{C^{\mathrm{ladder}}}{C^{\mathrm{ladder}} + C_p^S}[\phi]^T\Theta\Theta^T[\phi]y + [\phi]^T[F]f \tag{13}$$

or

$$\begin{aligned}&[m]\ddot{y} + [(k + k^{\mathrm{sc}}) + \Delta k]y\\ &\quad = [\phi]^T\left[\frac{C^{\mathrm{ladder}}}{C^{\mathrm{ladder}} + C_p^S}\Delta K\right][\phi]y + [\phi]^T[F]f\end{aligned} \tag{14}$$

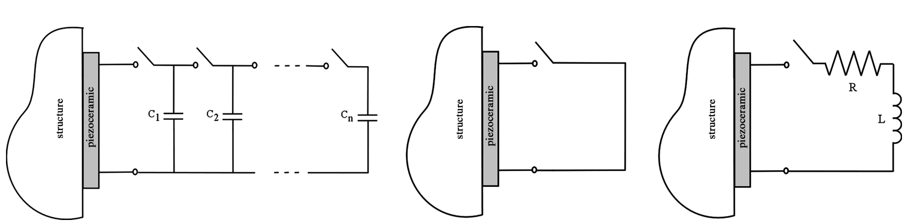


**Fig. 2 Piezoceramic actuators with switched shunts circuits: capacitance ladder, short circuit, and resistor/inductor.**

Note that by changing the capacitance in the shunt circuit in real time, the stiffness of the system can be varied in real time. Also, noting the similarities between the Type I mechanical spring modal equation of motion [Eq. (4)] and Eq. (14), it is clear that a piezoceramic actuator with a capacitance ladder shunt is a Type I piezoceramic spring.

### B. Type II Piezoceramic Spring: Switching Ground Shunt

When the switch to a ground shunt (short circuit) is shut, it can be shown [9] that $Q^{\text{app}}$ is equal to the negative of the generated charge produced by the actuator:

$$Q^{\text{app}} = -q_a = \Theta^T[\phi]y \tag{15}$$

and when the shunt switch is open, the applied charge is

$$Q_o^{\text{app}} = -q_{ao} = \Theta^T[\phi]y_o \tag{16}$$

where $q_a$ is the generated charge, and $q_{ao}$ and $y_o$ are the generated charge and the modal displacement vector at the time the shunt switch is opened, respectively. It is assumed that $Q_o^{\text{app}}$ remains constant while the shunt switch is open. Substituting Eqs. (15) and (16) into Eq. (10) yields

$$[m]\ddot{y} + [k + k^{\text{oc}}]y = s[\phi]^T \frac{1}{C_p^S}\Theta\Theta^T[\phi]y_o + (1-s)[\phi]^T\frac{1}{C_p^S}\Theta\Theta^T[\phi]y + [\phi]^T[F]f \tag{17}$$

or

$$[m]\ddot{y} + [(k + k^{\text{sc}}) + \Delta k]y = s[\phi]^T[\Delta K][\phi]y_o + (1-s)[\phi]^T[\Delta K][\phi]y + [\phi]^T[F]f \tag{18}$$

where $s$ switches between 0 and 1 depending on the position of the shunt switch:

$$s = 0 \rightarrow \text{Shunt Switch Shut}$$

$$s = 1 \rightarrow \text{Shunt Switch Open}$$

Again, noting the similarities between the Type II mechanical spring modal equation of motion [Eq. (8)] and Eq. (18), it is clear that a piezoceramic actuator with a switching ground shunt is a Type II piezoceramic spring.

It is interesting to note that the state-switching [3,4] vibration control technique uses a switching ground shunt. The system is kept in a high stiffness state (open circuit) until maximum/minimum modal displacement is reached. Then the system is switched to low stiffness state (short circuit) until the system returns back to its modal equilibrium point ($y = 0$), where it is returned to a high stiffness state. Because the shunt switch is opened at the modal equilibrium point, $y_o = 0$; therefore, a system using the state switching technique can be described by

$$[m]\ddot{y} + [(k + k^{\text{sc}}) + \Delta k]y = (1-s)[\phi]^T[\Delta K][\phi]y + [\phi]^T[F]f \tag{19}$$

Even though Eq. (19) is very similar to the equation for single modal control of a Type I system [Eq. (5)], it is clear that piezoceramic actuators using switching ground shunt circuits are really Type II systems.

### C. Type III Piezoceramic Spring: Switching Resistor/Inductor Shunt

The equation of motion for a system with a switching resistor/inductor shunt is very similar to the equation of motion of a Type II piezoceramic spring. However, because of the second order underdamped dynamics of the resistor, inductor, and piezoceramic actuator circuit, an applied charge greater than, but opposite to the generated charge can be achieved. (Note that this is not, strictly speaking, a pure variable stiffness approach because of the added dynamics of the shunt; however, one could consider it to be a frequency-dependent stiffness.) The maximum magnitude of this applied charge ($Q_*^{\text{app}}$) depends on the resistance and the inductance in the shunt circuit, the time at which the shunt switch is shut, and the time the shunt switch remains shut [9]. A system using a piezoceramic actuator with a switching resistor/inductor shunt can be described as

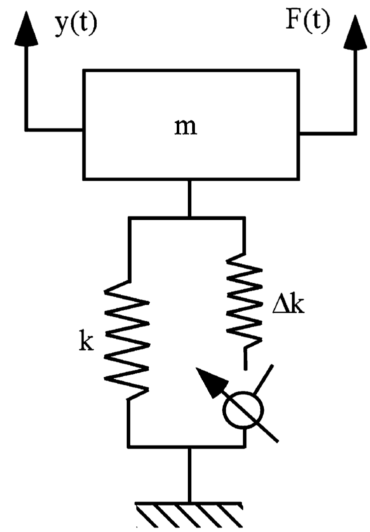


**Fig. 3 Example of a Type III modal system.**

$$[m]\ddot{y} + [k + k^{\text{oc}}]y = s[\phi]^T\Theta\frac{Q_*^{\text{app}}}{C_p^S} + (1-s)[\phi]^T\Theta\frac{Q^{\text{app}}}{C_p^S} + [\phi]^T[F]f \tag{20}$$

Using Eqs. (15) and (16), this can be rewritten as

$$[m]\ddot{y} + [k + k^{\text{oc}}]y = s[\phi]^T \frac{1}{C_p^S}\Theta\Theta^T[\phi]y_* + (1-s)[\phi]^T\frac{1}{C_p^S}\Theta\Theta^T[\phi]y + [\phi]^T[F]f \tag{21}$$

or

$$[m]\ddot{y} + [(k + k^{\text{sc}}) + \Delta k]y = s[\phi]^T[\Delta K][\phi]y_* + (1-s)[\phi]^T[\Delta K][\phi]y + [\phi]^T[F]f \tag{22}$$

Note the similarities of the modal equations of motion for Type II piezoceramic spring [Eqs. (18) and (22)]. They are identical except that now the magnitude of the charge applied to the piezoceramic actuator can be greater than the magnitude of the generated charge. In terms of a Type II mechanical spring, the secondary spring ($\Delta k$) is no longer just freed or clamped at $y_o$ [see Eq. (18)], but it can also be compressed or elongated such that the new clamping point, $y_*$, is greater in magnitude than $y_o$ (see Fig. 3). In essence, the resistor/inductor shunt circuit has changed the applied charge on the piezoceramic actuator making it appear to have a different clamped initial condition, $y_o$. Because this is a different type of mechanical spring than a Type I or II spring, it is designated as a Type III variable stiffness member.

## IV. Conclusions

In this paper, the similarities between mechanical springs and piezoceramic springs were presented. A Type I mechanical spring is similar to a piezoceramic actuator used with a capacitance ladder shunt circuit. It was also shown that a Type II mechanical spring is similar to a piezoceramic actuator used with a switching ground shunt circuit. A new mechanical spring type, Type III, was defined so that the piezoceramic actuator used with a switching resistor/inductor shunt circuit could also have a mechanical spring counterpart.

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